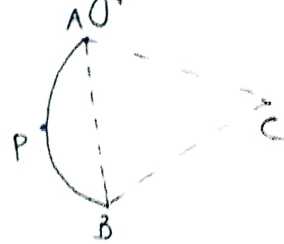
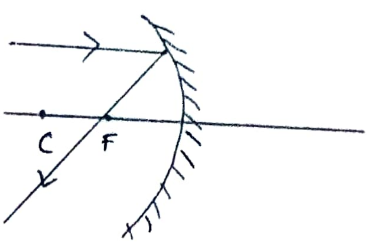
RAY OPTICS - 9

[Some terms associated with optics]

[Pole - It is the geometrical centre of a spherical mirror.
Centre of Curvature - It is the centre 'c' of the sphere of which the mirror forms a part.
Principal axis - the imagenary line passing through the pole and centre of curvature.

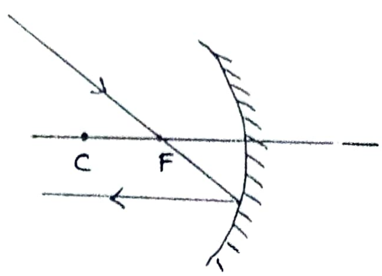


Lenear apeiture - It is the diameter AB of the circular boundary of the spherical mirror.
Angular aperture - It is the $\angle ACB$ subtended by the boundary of the spherical mirror at C.
Principal focus - A narrow beam of light parallel to the principal axis is either converges or appears to diverge from a point F on the principal axis. A concave mirror has a real focus while convex mirror has a virtual focus.
According to sign conventeon - A concave mirror has f and R negative and convex mirror has f and R positive.



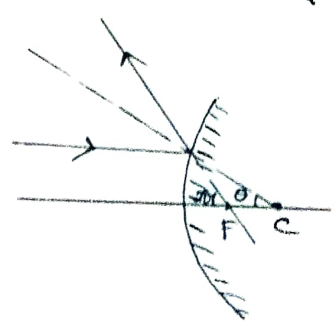
Concave mirror

A ray incident parallel thru' to the principal axis passes thru' the focus



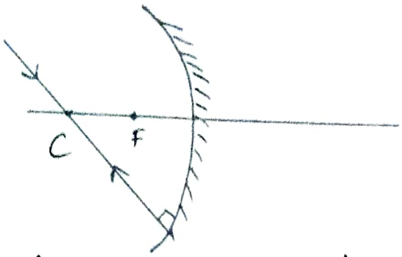
Concave mirror

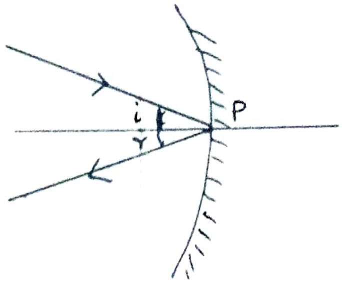
A ray passing thru' F after reflection emerge parallel to the principal axis.



Convex mirror

A ray incident on a convex mirror diverge after reflection. (It appears to come from F)

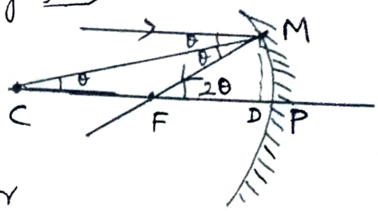




A ray passing thru' C or directed towards C (for convex mirror) falls normally ($\angle i = \angle r = 0$) simply retraces its path.

For any ray incident at any $\angle^{le}$ at the pole follows the laws of reflection.

Focal length – It is the distance b/w the focus F and the pole P of the mirror denoted by f.]



To prove $f = R/2$

Consider a ray parallel to the principal axis striking the mirror at M. Then CM ⊥ mirror at M. MD ⊥ principal axis Then $MCP = \theta$ and $MFP = 2\theta$.

$$\tan\theta = \frac{MD}{CD} \text{ and } \tan 2\theta = \frac{MD}{FD}.$$

For small θ, $\tan\theta \sim \theta$ and $\tan 2\theta \sim 2\theta$

$$\therefore \frac{MD}{FD} = 2\frac{MD}{CD}$$

$$\text{or } FD = \frac{CD}{2}.$$

ie for small aperture, D is close to P

$$\therefore FD = f \text{ and } CD = R$$

$$\therefore \boxed{f = R/2}$$ ⟹ Valid only for mirrors.

Mirror formula

Figure(1) shows the ray diagram for the image formation by a concave mirror when the object is kept beyond C.

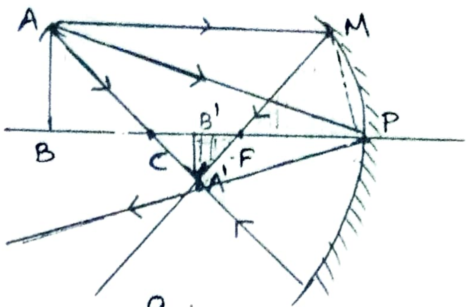


fig (1)

AB is the object of height 'h' and A'B' is a real inverted diminished image of height 'h'' formed b/w F and C. Consider the right angled triangles A'B'F and MPF, as they are similar;

$$\frac{A'B'}{MP} = \frac{B'F}{FP} \quad \text{or} \quad \frac{A'B'}{AB} = \frac{B'F}{FP} \quad (\because PM = AB) \longrightarrow (1)$$

Also right angled triangles A'B'P and ABP are similar

$$\frac{A'B'}{AB} = \frac{B'P}{BP} \longrightarrow (2)$$

[sign convention:
* All distances are measured from the pole of the mirror
* Direction of incident light is taken positive direction opposite to it is considered negative]

Comparing eqns 1 and 2

$$\frac{B'F}{FP} = \frac{B'P - FP}{FP} = \frac{B'P}{BP}$$

Applying sign conventions

$B'P = -v$; $FP = -f$

and $BP = -u$

[u - object distance
v - image distance
f - focal length]

Using these, $\frac{-v+f}{-f} = \frac{-v}{-u}$

$$\frac{v-f}{f} = \frac{v}{u}$$

Dividing thro' out by 'v' and rearranging,

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}.} \Longrightarrow \text{mirror formula or mirror equation.}$$

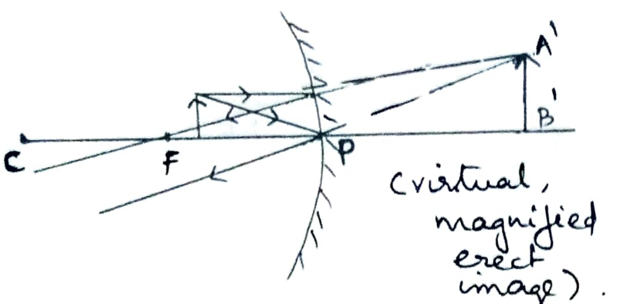
The ratio of height of image to the height of object is called linear magnification 'm'.

$$m = \frac{h'}{h} \; ; \quad \text{From fig : } \frac{A'B'}{AB} = \frac{B'P}{BP}$$

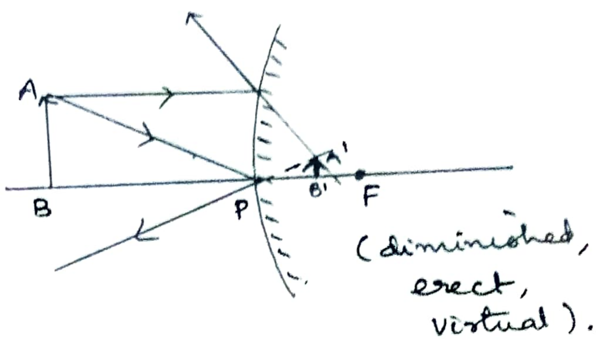
$$\frac{-h'}{h} = -\frac{v}{u}$$

$$\therefore \boxed{m = \frac{h'}{h} = -\frac{v}{u}}$$

Image formation by a (i) Concave mirror and (ii) Convex mirror with object b/w P and F.



(i)



(ii)

* laws of reflection is true for all points of the surface of the mirror. Image will be visible even if half of the mirror is covered with an opaque material.
* As the area of reflecting surface is reduced, the intensity will be reduced.

REFRACTION : A ray of light incident obliquely at an interface separating two media (different optical density) deviates from its path in the 2^{nd} medium. This phenomenon is called refraction of light. (Laws of refraction - refer text).

Snell's law of refraction - The ratio of sine of angle of incidence (i) to the sine of angle of refraction (r - in 2^{nd} medium) is a constant - which is refractive index of 2^{nd} medium w.r.t 1^{st} medium

ie. $$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

→ n_{21} depends on the wavelength of light but is independent of ∠^{le} of incidence.

→ If $n_{21} > 1$ ie $r < i$ → refracted ray bends towards the normal or medium ② is optically denser than medium ①.

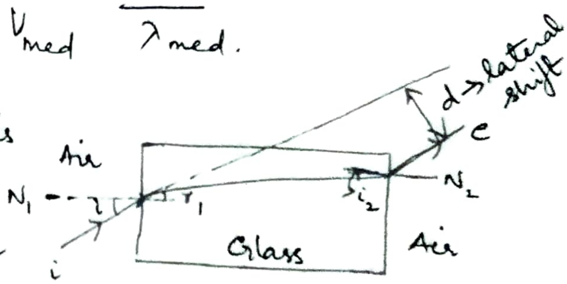
→ If $n_{21} < 1$ ie $r > i$ → Refracted ray bends away from the normal.

→ $$n_{21} = \frac{1}{n_{12}}$$

→ $$n_{32} = \frac{n_{31}}{n_{21}} = n_{31} \times n_{12}$$

→ Absolute ref. index $n = \frac{c}{v_{med}} = \frac{\lambda_{vac}}{\lambda_{med}}$.

Refraction through a glass slab

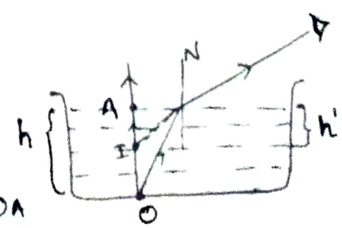


Here $r_2 = i_1$ ie emergent ray is parallel to the incident ray or there is no deviation but the ray undergoes lateral shift.

* The bottom of a tank filled with water appears to be raised due to refraction.
* The perpendicular distance b/w the incident ray and the emergent ray — lateral shift given by

$$d = \frac{t}{\cos r} \sin(i - r).$$ [t – thickness of glass slab)

Real and apparent depth



Refractive index of medium,

$$n = \frac{\text{Real depth}}{\text{Apparent depth.}} = \frac{h}{h'} = \frac{OA}{IA}$$

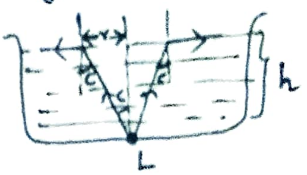
* The distance OI through which an object appears to shift is called normal shift ie

$$d = OA - IA$$

$$d = t\left[1 - \frac{1}{n}\right]$$

* higher the value of n, greater is the normal shift.

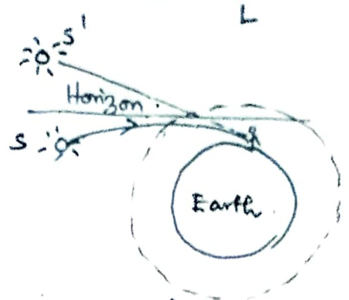
* A light source kept at the bottom of a tank filled with liquid (water) will have the light spread along the area of a circle of radius 'r' given by:



$$\tan C = \frac{r}{h}$$

or $r = h\tan C$

where 'C' is the critical $\angle^{le}$ of the medium given by $n = \frac{1}{\sin C}$



or $\sin C = \frac{1}{n}$

$\therefore$ Area $= \pi r^2 = \pi (h \tan C)$.

* Apparent position of the sun ie sun is visible a little before the actual sunrise and little while after the sunset (approx 2mins) due to atmospheric refraction.
The apparent flattening of the sun at the time of sunrise and sunset is also due to the same reason.

* The density of and ref. index of air layer decrease with altitude. The light rays from sun travel from rarer to denser, hence bend more and more towards normal. The apparent shift in the direction of sun is about $\frac{1}{2}^\circ$ which corresponds to time of 2 minutes.
[one full rotation $24h \Rightarrow 360^\circ$

$\therefore 1^\circ \Rightarrow 4$ mins

& $\frac{1}{2}^\circ \Rightarrow 2$ mins].

Total Internal Reflection (TIR).

When light travels from an optically denser medium to a rarer medium, at the interface, it is partly reflected back into the same medium. This is known as internal reflection. Under certain condition, whole of the light can be made to reflect back into the ~~same~~ denser medium. This is known as total internal Reflection (TIR).

* when a ray travels at a small angle of incidence from a denser to rarer medium, the refracted ray bends away from the normal. As $\angle i$ increases, $\angle r$ also increases. Then for a particular $\angle^{le}$ of incidence, the $\angle^{le}$ of refraction is $90°$, ie the refracted ray grazes the interface. The angle of incidence in the denser medium for which the angle of refraction is $90°$ is called the critical angle of the denser medium denoted by i_c!
If $i > i_c$ then no light is refracted, but whole of the it is reflected back into the denser medium — known as TIR. Conditions for TIR to occur.
* light must travel from denser to rarer medium.
* $\angle i > \angle i_c$.

$$\frac{\sin i}{\sin r} = n_{12} = \frac{1}{n_{21}}$$

when $i = i_c$; $r = 90°$

$$\therefore \sin i_c = \frac{1}{n_{21}}$$

or $$\boxed{n_{21} = \frac{1}{\sin i_c}}$$

* ref. index of the denser medium w.r.t rarer medium is the reciprocal of the sine of critical angle denser medium.
(ie smaller $i_c \to$ larger n)

Applications:
Optical fibres : They are extensively used in transmitting audio or video signals through long distance without much loss of energy. Principle of transmission of such based on TIR. They are fabricated with high quality glass/quartz fibres — of diameter μm range. Each fibre consists of an inner coating of high ref. index (low i_c) called core and outer coating of low ref index called cladding. when light at one end enters the fibre at a suitable angle of incidence, it undergoes

repeated TIR along the length of the fibre.
(Refer T.B. for fig) and finally comes out at the other end. Since it undergoes TIR inside the fibre there is minimum loss in the intensity of light signal. They are so fabricated that light entering at one end will be incident greater than i_c. Even if the fibre is bent, light can easily pass through the entire length. A bundle of optical fibres act as optical pipe.

→ Used for transmitting and receiving electrical signals which are converted to optical signals by transducers in the field of communication. (Upto 2000 signals managed)

→ In Medical diagnosis – light pipes facilitate visual examination of internal organs – endoscopy.

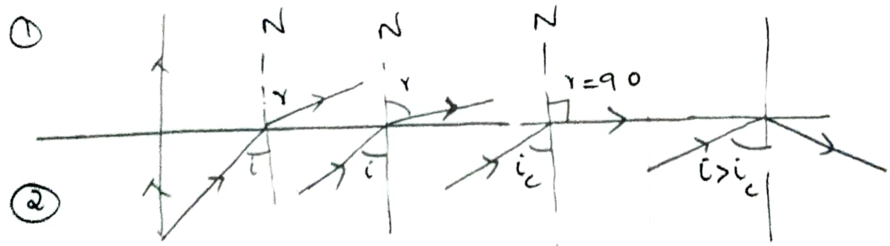


figure showing phenomenon of TIR for $i > i_c$.

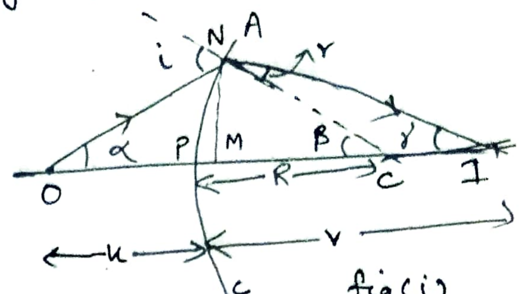
<u>Some other applications of TIR</u>

(i) Mirage – optical illusion that occurs in deserts (on coal tarred roads also) on hot sunny days. An object (say tree) appears inverted to a distant observer as if it were on the bank of a pool of water. On hot days, the sand/land gets hotter and air near to the surface becomes hot and rarer ie ref. index of layers of air goes on increasing with height. A ray of light from the top of a tree goes down from denser to rarer medium and keeps bending away from the normal. At a particular layer, when $i > i_c$

consider small aperture of the surface, NM = NP.

from fig $\tan\alpha = \frac{MN}{OM}$; $\tan\beta = \frac{MN}{MC}$

$\tan\gamma = \frac{NM}{MI}$



fig(i)

for ΔNOC, $\angle i$ is exterior $\angle^{le}$.

$\therefore \angle i = \angle\alpha + \angle\beta$.

or $i = \alpha + \beta$.

$$i = \frac{MN}{OM} + \frac{MN}{MC} \quad \text{---(1)}$$

Similarly $r = \angle NCM - \angle NIM$)

$r = \beta - \gamma$.

ie $$r = \frac{MN}{MC} - \frac{MN}{MI} \quad \text{---(2)}$$

By snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad \text{---(3)}$$

$n_1 \sin i = n_2 \sin r$

for small i and r.

$n_1 i = n_2 r$.

$$n_1\left(\frac{\cancel{MN}}{OM} + \frac{\cancel{MN}}{MC}\right) = n_2\left(\frac{\cancel{MN}}{MC} - \frac{\cancel{MN}}{MI}\right) \quad \text{---(4)}$$

Rearranging Eqn (4)

$$\frac{n_1}{OM} + \frac{n_1}{MC} = \frac{n_2}{MC} - \frac{n_2}{MI}$$

Applying sign conventions,

$OM = -u$; $MC = +R$; $MI = +v$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Therefore for light incident on a spherical surface such that it travels from rarer to denser medium

$$\frac{n_2}{\text{image distance}} - \frac{n_1}{\text{object distance}} = \frac{n_2 - n_1}{\text{Radius of curvature}}$$

TIR takes place and the reflected ray reaches the eye of the observer. To the observer it appears to come from point-I. Hence he sees an inverted image of the tree which create an impression as if it were reflected by a pool of water as in fig (i).

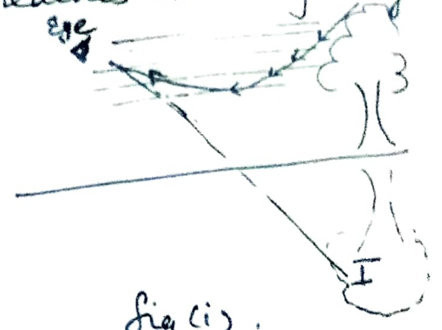
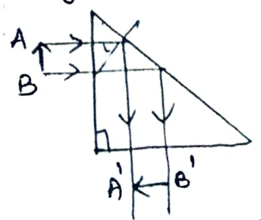


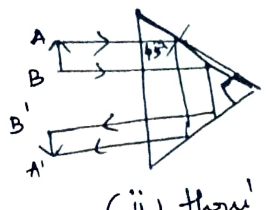
fig (i).

(ii) Brilliance of diamonds. – The 'n' of diamond w.r.t air is 2.42. The critical angle for diamond–air interface is 24.4°. Diamond is cut suitably so that light entering the diamond undergoes TIR repeated at various faces and emerges out in diff. directions → gives the sparkling effect.

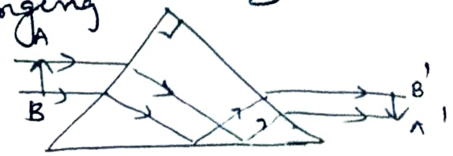
(iii) Total reflecting prism → right ∠led isosceles prism (ref. index 1.5 and $i_c = 42°$) – used as reflector to deviate light by 90° or 180° and to produce ~~erected~~ its size. image of an object. without changing



(i) thru' 90°



(ii) thru' 180°

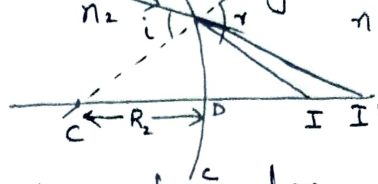


Used in binoculars and submarine periscopes. (The intensity of these images will be greater that that formed by using plane mirrors).

Refraction at Spherical Surfaces.

The fig. shows image formation by a point object O at I for a spherical surface of radius of curvature R. The ray is incident from a medium of ref. index n_1 into another of ref. index n_2.

Similar procedure can be applied to the interface ADC when light travels from denser to rarer medium as shown in the figure (And the eqn is given by interchanging n_1 and n_2)

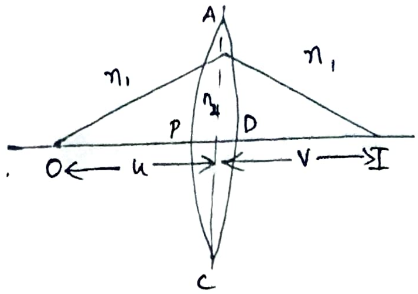


$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$$

fig (ii)

Refraction by a lens – Lens Makers formulas.

Consider 'L' to be a thin lens of refractive index n_2 and it is placed in a medium of ref. index n_1. Fig (i) ~~shows~~ and (ii) shows the image formation by the two surfaces APC and ADC (convex and concave) of radii of curvature R_1 and R_2 respectively. Fig (iii) shows the combination of the two or the refraction by a thin lens.



For the 1st surface APC as shown in fig (i), the eqn is (the image is formed at I' which is virtual).

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \text{—(1)}$$

for the 2nd surface ADC as shown in fig (ii), the 1st surface acts as a image formed by the 1st surface acts as a virtual object and final image is formed at I

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_1 - n_2}{R_2} \quad \text{—(2)}$$

Adding (1) and (2)

$$\cancel{\frac{n_2}{v'}} - \frac{n_1}{u} + \frac{n_1}{v} - \cancel{\frac{n_2}{v'}} = n_2 - n_1 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$n_1 \left[\frac{1}{v} - \frac{1}{u}\right] = n_2 - n_1 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{n_2 - n_1}{n_1}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$\left\{ \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \right.$ by thin lens formula.

$$\frac{1}{f} = \left[\frac{n_2}{n_1} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boxed{\frac{1}{f} = \left[n_{21} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

$\Rightarrow$ known as lens makers formula.

Sign convention : (i) for Convex lens R_1 is $+ve$ R_2 is $-ve$ and f is positive $\rightarrow$ converging when $n_2 > n_1$

(ii) for Concave lens R_1 is $-ve$ and R_2 is $+ve$ and f is negative. $\rightarrow$ diverging for $n_2 > n_1$

$\Rightarrow$ By the above formula, a lens will be converging when $n_2 > n_1$ and diverging when $n_2 < n_1$. where n_2 is the ref. index of lens and n_1 ref. index of surrounding medium.

$\Rightarrow$ For equi convex or equi concave lens $R_1 = R_2 = R$.

$\Rightarrow$ Focal length of a lens increase when a lens is immersed in a liquid of refractive index n_l, because.

$$\frac{1}{f} \propto (n_{gl} - 1)$$

Also $n_{gl} = \frac{n_g}{n_l}$ which is less than n_{ga} (glass w.r.t air).

$\Rightarrow$ A convex lens can be a diverging lens when $n_g < n_l$

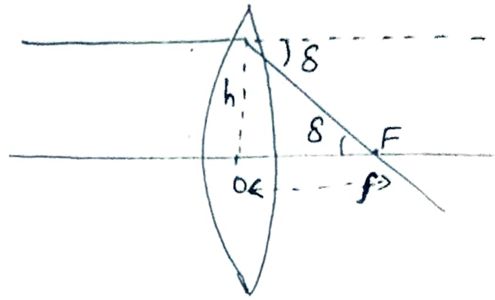
①

<u>Power of a lens</u> is a measure of convergence or divergence produced by a lens in the light falling on it. It is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from optical centre.

$\tan\delta = \frac{h}{f}$

if $h = 1$; $\tan\delta = \frac{1}{f}$

or $\delta = \frac{1}{f}$ (for small values of δ).



$\therefore P = \frac{1}{f}$

SI unit of power of a lens is dioptre (D)

$1D = 1m^{-1}$.

Power of a lens is positive for a converging lens and negative for a diverging lens.

<u>Combination of thin lenses in Contact</u>

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other.

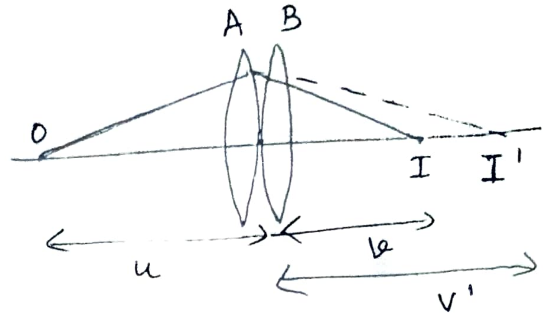
Let a point object 'O' be placed beyond the focus of lens A. 'A' produces an image at I'. Since image I' is real, it serves as a virtual object for the second lens B, producing the final image at I. (Since the lenses are thin, optic centres are coincident).

For the image formed by first lens A,

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \quad \text{—— (1)}$$

For the image formed by the second lens B,

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad \text{---} (2)$$



Add (1) and (2)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the two lens system is considered equivalent to a single lens of focal length f, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

* The above eqn is valid for any no. of lens. Hence the effective power of combination will be

$$\boxed{P = P_1 + P_2}$$

* The total magnification of the combination is

$$m = m_1 \times m_2 .$$

Dispersion by a prism

The phenomenon of splitting of light into its component colours is known as dispersion. The pattern of colour components of light is called the spectrum of light.